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THE PURE THEORY OF IMPURE PUBLIC EXPENDITURES RECONSIDERED: A NOTE ON THE PUBLIC AND MERIT CHARACTERISTICS’ APPROACH*

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The paper reconsiders the implications of the public characteristics’ approach in order to formulate a rigorous enlarged social welfare theory allowing to analyse impure public goods and merit wants. We treat externalities as “objective characteristics” of commodities building upon the contributions of Lancaster [1976] and Auld and Eden [1990]. Focusing on Lancaster’s original simple specifications, we examine how the properties of collective goods are modified and pseudo-demands for impure public goods are derived. Our analysis shows the equivalence between public provision and subsidy solutions. Considering the Auld-Eden analysis of ‘aggregate externalities’, we point out the differences of this approach and eliminate an ad hoc assumption of irrational behaviour. We also formulise Musgrave’s merit argument as a coercivity phenomenon. Following the Italian tradition, we propose a simple framework in which the “multiple preferences” representation of merit wants arises naturally. In this way we partially vindicate the treatment of Pazner [1972] and Roskamp [1975] as well as Cosciani’s tutorial model, providing a justification for different utility functions of citizens and governors. Finally we show how the Auld-Eden model may be reinterpreted as a merit wants’ analysis.

I. INTRODUCTION

Although the concept of public good has become a less accurate tool, it is still considered quite relevant in relation to the economic theory and its social applications. With collective consumption viewed as an “extreme polar case”, even by P.A.

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* I want to thank all those who personally discussed with me some points connected to this subject, and in particular Richard and Peggy Musgrave, Richard Tresch, John Hey. I am also very obliged for the helpful comments of an anonymous referee. A preliminary version of this paper has been discussed at the Scientific Conference of the Italian Society of Public Economics, SIEP, in November 1991 in Pavia. The author only is responsible for all possible errors and opinions expressed in the paper. Financial support from the MURST is gratefully acknowledged.

1 Problems of great importance and actuality – like the environmental ones – are inextricably connected with the theory of public goods and factors. For an opposite view cf. De Clercq [1985] and Block [1989].
Samuelson [1955], the analysis of some kind of mixed goods has been attempted by various authors, who have taken into account different degrees of ‘indivisibility’ and rivalness. For example, the theories of mixed goods (with Musgrave) and impure public goods (with De Serpa) try to address the questions of optimal provision and utilisation of public goods when private benefits or ‘generic congestion’ result from their use.\(^2\) However, these theories represent a direct development of Samuelson’s idea of individual consumption of public good, which has been criticised because it is not observable and “the applicability of public goods reasoning is not dependent on any observed condition on these variables, in particular not on their observed equality”.\(^3\) This tool, technically applied for instance by Milleron [1972], is also connected with the analogy between joint supply and collective consumption goods, pointed out by Samuelson.\(^4\)

This approach may instead represent not only a useful devise, but also (once goods are properly defined) a possible representation of the general problem even through the characteristics analysis, as proposed for instance by Lancaster [1976].\(^5\) It also allows to take properly into account quite complicated relationships between public (or meritorious) characteristics and goods and to formulate a correct treatment of many impure public goods, as shown by Auld and Eden [1990]. In fact one may feel that even now after so many years, a little effort is still needed to increase the rigour of the analysis especially in some applications of the concept of public goods (for instance to environmental problems), where someone finds out that, instead of the mixed good, the “merit good is on the border line between a private and a pure public good”.\(^6\)

\(^2\) Following J. Rothenberg [1970] we assume that ‘generic congestion’ subsumes both phenomena of pure congestion and pollution as well as any possible combination of the two. For DeSerpa [1978] in the theory of impure public goods the “assumption of indivisibility is explicitly dropped and aggregate utilization replaces the discrete number of shares as an index of congestion” (p. 68). One of the first analysis is due to Oakland [1972], followed by James [1974], Sandler [1975]. On a related topic cf. also Haveman [1973].

\(^3\) Bradford [1971] also claims that “it is unfortunate that they were introduced by Samuelson at the very foundation of the analysis” (p. 1122). See also Vagliasindi [1991].

\(^4\) “Government supplies products jointly to many people”, Samuelson [1955, p. 355]. However, he did not seem completely satisfied of this hint, because it may result in an incorrect analogy and so he added: “It is sometimes thought that increasing the number of citizens who are jointly supplied with public goods leads to a similar determinate result. This is a reasoning from an incorrect analogy”. See also Samuelson [1969].

\(^5\) One of the most innovative approaches to public goods, presented in Sandmo [1973] and more recently in Auld and Eden [1990], is based on the characteristic analysis of goods developed by Lancaster and Muth also called household production theory because it introduces production into the utility functions.

\(^6\) “Exclusion is technically feasible ... and there is some rivalry in use ... Rivalry in use may give rise to congestion. Thus between the polar case of pure private and pure public goods we may have many intermediate forms”, Siebert [1987, pp. 62-3]. We may find many more examples which support De
In what follows the public characteristics’ approach will be critically re-examined, building upon Lancaster’s contribution and the recent paper by Auld and Eden [1990]. In Section II, focusing on simple specifications, it will be shown in which way the properties of Samuelson’s collective goods are modified and how the pseudo-demands for impure public goods should be derived. Then, in Section III, the merit (or demerit) argument too will be formalised in the same setting. This helps to understand its nature and to discuss a greater part of it in the framework of welfare economics. However, it is beyond our aims to give a systematic picture of these problems and to offer global visions. Instead, in what follows further theoretical problems and issues will come up more than definite solutions and answers.

II. LANCASTER’S APPROACH TO PUBLIC CHARACTERISTICS

II. A. The Original Lancaster Impure Public Goods Model

In 1976 Lancaster applies his consumption theory to public goods, allowing to treat externalities “as objective characteristics of the good, rather than as properties of individual preferences” and “to separate the technical specification ... from the subjective aspect”. This contribution seems to go in the direction previously proposed. Hence, it is fundamental to re-examine critically this work and in particular the very simple two goods’ model specification, which introduces the “impure public good” $X$ that has both a (unit of) public characteristic $X$ and a (partial) private one $aX$. The private characteristic $z_i$ for individual $i$ is the sum of the private good $x_i$ and the private characteristic $aX_i$ of the impure good consumed by him. The technical specification (of the household production function) is simply reduced to the constant ‘$a$’ (the amount of private characteristic per unit of impure good); congestion is ruled out; the usual well behaved transformation function $T$ is assumed; each individual utility depends on the total amount of public characteristic $Z$ and the impure good is “supplied by the government as a public good” in the amount $X_G$. The specification of the model is:

(1) \[ u^i = u^i(z_i, Z) \]  \hspace{1em} (i = 1, \ldots, n) \hspace{1em} (8 - 1)

(2) \[ z_i = x_i + aX_i \]  \hspace{1em} (i = 1, \ldots, n) \hspace{1em} (8 - 2)

(3) \[ Z = \sum_{i=1}^{n} X_i + X_G \]  \hspace{1em} (8 - 3)

(4) \[ T' = T \left( \sum_{i=1}^{n} x_i; \sum_{i=1}^{n} X_i + X_G \right) \]  \hspace{1em} (8 - 3)

Clercq’s pessimistic terminological view, even if the case of “mixed goods” proposed by R. Musgrave is neatly defined (as well as the one of merit goods).

7 Cfr. p. 129. The same analysis is reported in Lancaster [1990]. Another interesting paper is Cornes and Sandler [1986]. For applications to welfare measurement cfr. Bockstael and McConnel [1983], Morey [1984] and [1985].
On this basis (assuming an interior solution) Lancaster is able to derive the marginal rate of substitution (MRSG between $X_i$ and $x_i$ and MRSC between characteristics (or $X_{C_i}$ and $x_i$), with \( MRSG = a + MSRC = a + u_2/u_1 \) that characterises the first best optimal solution (OPT) of the model, the private one (PRIV, where the impure public good is allocated by a perfectly competitive private market) and the public one (PUB, where the impure public good is allocated by the government) and to compare them, assuming that the transformation curve $T$ is linear.

\[
\begin{align*}
(5) \quad & T_2/T_1 = MRSG_i = MRSG_j \quad \text{(PRIV)} \\
(6) \quad & T_2/T_1 = \sum_{i}^{n} MRSC_i \quad \text{(PUB)} \\
(7) \quad & T_2/T_1 = a + \sum_{i}^{n} MRSC_i = \sum_{i}^{n} MRSG_i - (n-1)a \quad \text{(OPT)}
\end{align*}
\]

It is easy to demonstrate that, in comparison with the optimal solution, the private and the public ones "will have every individual consuming more of the private, and less of the mixed good". In fact:

\[
(8) \quad \sum_{i}^{n} MRSC_i(PUB) = a + \sum_{i}^{n} MRSC_i(OPT) > \sum_{i}^{n} MRSC_i \quad \text{(OPT)}
\]

**FIGURE 1**

Lancaster concludes that, in the case of impure public goods, optimality requires supply through the market with a public subsidy-scheme.

In order to examine graphically the optimal solution under the hypotheses of a linear characteristic transformation curve CTC (whose slope is $t = T_2/T_1 - a$) we
can simply use the Samuelson diagram, re-formulated with the private $z$ and public $Z$ characteristics and two individual (a and b), assuming an interior solution like $E$, where $t = \sum_1^n \text{MRSC}_i$.

The similarity to the case of mixed goods is noteworthy. One may even think that the impure public good of Lancaster is a sort of mixed good re-formulated in terms of characteristics' analysis and its technical language. However, this is not completely true, because one cannot speak any longer of public and private goods (but only of public and private characteristics) and also because Lancaster correctly formulates the subsidy solution in a different way, as shown in what follows.

II. B. The Auld-Eden Approach to Aggregate Externalities

The Auld-Eden characteristics' approach to 'aggregate externalities', represents a generalisation of the original aggregate consumption externality model due to Tresch [1981].

The case taken into consideration seems more general than the one considered previously following Lancaster, even if they use partial equilibrium analysis and consumers have "identical quasi-concave utility functions" (so that the authors drop the consumers' index from the utility function, goods and characteristics). In this way, one can consider $n$ consumers, $J$ goods ($X_j$) and $k$ characteristics ($K_l$), even if they concentrate on the case of what Lancaster would have called two "impure goods" and three characteristics (1 and 2 are private, while $k$ is public and negative) in order to give a nice geometrical interpretation.

Following Auld and Eden we may state the consumers' problem in general terms as:

\[
\begin{align*}
\max_{X_j} u &= u(K_{l1}^1, K_{l2}^1, \ldots, K_{lk}^1) \\
K_l^i &= \Sigma_j b_{lj} X_j^i \\
M &= \Sigma_j p_j X_j^i
\end{align*}
\]

From the private maximisation problem the following f.o.c. (original eq. (4)) holds:

\[
\text{MRSC}_{1,2}^i = u_1/u_2 = -dK_2/dK_1 = \text{MRTC}_{1,2} \quad \text{for } i = 1, \ldots, n \quad (4)
\]

One the other hand the f.o.c. of the social optimum (original eq. (5)) is:

\[
\begin{align*}
u_1/u_2 &= -dK_2/dK_1 - (d\Sigma_i K_{l1}^i/dK_1)\Sigma_i u_{ki}/u_2 = \\
\text{MRTC}_{1,2} + \text{MRTC}_{1,k} \Sigma_i \text{MRSC}_{k,2} \quad \text{for } i = 1, \ldots, n
\end{align*}
\]

Hence, in order to implement the Pareto optimum it is sufficient to eliminate the wedge between the two slopes through the usual Pigouvian tax-subsidy scheme. In
practice, as the wedge does not vary across individual households we may just ade-
quately tax the high-polluting good or subsidise the low-polluting good. Tresch’s
result holds substantially even in this more general model, while it was not verified
in the simple Lancaster model. Let us examine the relevant differences between the
two approaches.

First of all, differently from Lancaster the maximisation problem is solved in terms
of marginal rate of substitutions and transformations between characteristics and not
between goods (MRSG\(^i\)\(_{1,2} = T_2/T_1\)). Second, following Tresch’s ad hoc behavioural
assumption each household ignores the effects of his choice on the aggregate level
of pollution (\(\sum_i K^i_k = \sum_i \sum_j b_{kj} X_j^i\)). Naturally this is absurd in the basic two
consumers’ model being \(\sum_k K^i_k = 2K_k\) in this case clearly \(K_k\) cannot be small with
respect to the aggregate level of pollution, while it may be acceptable only as \(n\) tends
to infinity. Hence, the policy rule may represent a pretty good approximation with a
large number of consumers.

However, the point we want to emphasise is more theoretical than practical, taking
the aggregate level of pollution \(\sum_i K^i_k\) as exogenous we are following a dangerous
ad hoc simplification. In general, it would mean that consumers externalise one direct
effect of their own action (consuming \(X_1^i, X_2^i\)) and it would imply a quite irration-al
behaviour, which is not acceptable on any methodological grounds. Should a consumer
disregard the way in which the acquisition of an additional unit of a non-excludable
public good would affect his own welfare only on the ground that it would also affect
other consumers?

Clearly this improvement would not substantially change the results, as individuals
have identical utility functions and income. The equal tax or subsidy on each good (as
stated by Tresch) will continue to hold in the Auld-Eden model, even if we correctly
assume that each individual \(h\) wedge between the two slopes is MRSC\(_{1,2}\) \(\sum_{i \neq h}\)
MRSC\(_{k,2}\) (for \(i\) different from \(h\)). Furthermore, the geometrical representation does
not need to be modified.

But in general, with different individuals in order to implement the Pareto optimum
we need different individual taxes and subsidies. In any case if each consumer cor-
rectly reveals his preference for any “impure good” the subsidy scheme proposed by
Auld and Eden would result in too high taxes and subsidies, as each household already
considers all the consequences of his choices on the aggregate level of pollution.

Given the previous considerations, to better explore, in a simple framework, the
consequence of the rejection of the ad hoc behavioural assumption that every consumer
ignores his effects on the aggregate level of the public characteristics we choose to
go back to the original Lancaster model.
II. C. A General Graphical Approach to Public Characteristics

In what follows starting from the Lancaster model we will describe a generalised graphical approach to public characteristics and compare it with the usual solution which – probably rising from Musgrave’s mixed goods – informs Tresch’s original aggregate consumption externality model and is present in its substance even in the recent and relevant extension of the model due to Auld and Eden. In the usual scheme each individual seems to formulate separately a demand for the private characteristic and a pseudo-demand for the public characteristic, which are separately aggregated (into D and PD) and after vertically added in order to find the amount of the subsidy. It is true that the argument \( X_i \) enters two times in the utility function \( u^i(x_i + aX_i, \sum_j X_j) \) of each individual \( i \) and that he has two benefits from his consumption of \( X_i \). However, there is no reason for each individual \( i \) to distinguish the private and social benefits due to \( X_i \). Furthermore, nothing assures us that he will pay only for his private benefit \( u^i \) and thus that an equal subsidy would solve the problem.\(^8\) In reality, a rational consumer does not ignore the consequences of his own economic activity on the utility function that he maximises, and he does not deliberately disregard any effect on his welfare (even if it is an unintentional product or it has an incidental character as Mishan puts it) of the variable \( X_i \) that is under his complete control. In fact for Lancaster each agent exactly values his own full benefits connected with his consumption of good \( X_i \) in his private demand \( D^i \). Thus only a pseudo-demand of other agents for the public characteristics of his consumption \( PD_{j \neq i}(X_i) \) should be added, so that the subsidy should be equal to this valuation. This is clearly the most general solution which may encompass the previous one as a special case. In fact: (1) it allows for different subsidies for each \( X_i \) and (2) we may also sum for each \( X_i \) the demand for private characteristic with the aggregate pseudo-demand for the public characteristic \( \sum_j PD^j(X_i) \).

The two solutions are represented in Fig. 2, where \( CM_X = t - a = T_2/T_1 \) is the marginal cost of \( X \) (in terms of the private good). In practice in the general graphical approach (the right hand graph of Fig. 2) we are representing two consumers \( a \) and \( b \) who already consider the full benefits, they receive from their consumption, in their demands \( D^a \) and \( D^b \). To reach the Pareto optimum the benefits that agents \( a \) and \( b \) receive from the consumption of the impure public goods made by the other – that is respectively \( PD^a(X_b) \) and \( PD^b(X_a) \), the pseudo-demands of \( a \) (\( b \)) for the impure

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\(^8\) Public intervention with subsidies may probably lead to an oversupply of the public good, being some public benefit evaluated twice in private and public calculations. In their treatment, like in Tresch [1981], it is explicitly assumed that each individual “ignores the marginal external effect of his consumption”, which is either an approximation or a hypothesis of irrational behaviour. One cannot strictly speak of external effect because the effect is on the very same agent who consumes; thus following the usual argument he should internalise the externality.
public goods consumed by $b$ (a) – should be taken into account. By the vertical sum of $D^a$ and $PD^b(X_a)$, we will obtain the social demand $D + PD(X_a)$, for the impure public goods consumed by $a$. Finally by horizontal summation of the social demands $D + PD(X_a)$ and $D + PD(X_b)$ we get the aggregate social demand $(P + DP)^{a+b}$ for the impure public good. In general the unit subsidies to $a$ and $b - PD^a(X_b)$ and $PD^b(X_a)$ – may differ and the Pareto optimum would not be reached with a general subsidy to the consumption of the impure public good.

**FIGURE 2**

Lancaster shows that in the general case the State may implement the optimal solution subsidising each $X_i$ by different amounts. The previous analysis clearly shows that each $X_i$ is a public good. Thus we are left with $n$ different Samuelson’s public goods. One can argue that it is not rigorous to speak of a “private” demand of $X$, because $X_i$ is a public good (as Samuelson may claim and the previous analysis shows) so it may be better to speak only of pseudo-demands. However, one can argue that the subsidy-scheme should be analysed, as suggested by Samuelson, in a game theoretical setting.

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9 The peculiarity of the usual solution (due to Tresch [1981]) – “much more amenable to corrective public policy action” -- does not depend on the technical hypothesis but on the previous ‘ad hoc’ consumers’ behaviour, which allows to simplify the model. We feel that even if it may represent a valuable and viable possibility from a practical point of view (as well as in real word applications pointed out by Lancaster himself) it is not an acceptable solution in a theoretical model.

10 In any case strategic behaviours do not imply that one ignores the internal consequences on the utility function of his own consumption, but instead that he also considers the reaction to the external effects caused by his activities. This does not imply that we entirely go back to Samuelson; because, even if his general formulation may cover the case in an analytical fashion, the impure good $X$ is not a collective consumption good and each individual consumption $X_i$ (or its public characteristic content) is not a good in the original approach. An interesting debate related to the present subject, between Samuelson and Musgrave, is contained in Margolis and Guiltton [1969] (eds.).
One may want to extend the analysis allowing for \( n \) consumers, two "impure goods" and three characteristics (1 and 2 are private, while 3 is public and negative) in order to compare geometrically the present approach to public characteristics with the model due to Auld and Eden. Assuming an internal solution, let us consider \( d \) the "private" demand for characteristic 2 and let \( m_2 = dK_2 - dK_1 \) and \( e_2 = d\sum_i K_i^i / dK_2^i \) represent respectively the implicit marginal cost of characteristic 2 in terms of characteristic 2 and the marginal rate of transformation between aggregate public characteristic and characteristic 2. Following the authors we will assume that characteristic 2 is the one associated with higher pollution \( e_2 < 0 \).

From the private maximisation problem of consumer \( i \) we can derive the following f.o.c.:

\[
(12) \quad \frac{u_2^i}{u_1^i} = MRSC_{2,1}^i = -dK_1 / dK_2 - (d\sum_i K_i^i / dK_2^i)u_k^i / u_1^i = m_2 + e_2MRSC_{k,1}^i
\]

while the f.o.c. of the social optimum is:

\[
(13) \quad \frac{u_2^i}{u_1^i} = MRSC_{2,1}^i = -dK_1 / dK_2 - (d\sum_i K_i^i / dK_2^i)\sum_i u_k^i / u_1^i = m_2 + e_2\sum_i MRSC_{k,1}^i
\]

Remembering that in Auld and Eden the private demand for characteristic 1 should instead satisfy:

\[
(10') \quad \frac{u_2^i}{u_1^i} = MRSC_{2,1}^i = -dK_1 / dK_2 = m_2
\]

we may draw Fig. 3 which compares the two solutions and that is analogous to Fig. 2 (while the demand for characteristic 1 would have been exactly similar to Fig. 2). The

**FIGURE 3**

only differences are that: (i) now demands "d" and "sc" are associated to characteristic 2 (and not to the impure public goods) and (ii) we are dealing with a negative public
characteristic so that pseudo-demands are negative and represent social costs (to be subtracted from the private demand; \(-e_2 \text{MRSC}^i_{k,1} < 0, \text{being } u^i_k / u^i_1 < 0\)).

In the right hand graph of Fig. 3 we depict the general graphical approach considering two consumers (a and b) whose demands \(d^a\) and \(d^b\) reveal the net benefits they receive from their consumption of characteristic 2. To reach the Pareto optimum the costs due to the consumption of the impure public goods made by the other agent – respectively \(sc^a(K^a_2)\) and \(sc^b(K^b_2)\) – should be explicitly taken into account. Subtracting vertically \(sc^b(K^b_2)\) from \(d^a\) we get the social demand \(d - sc(K^a_2)\), for the characteristic 2 consumed by a. Summing horizontally the two social demands \(d - sc(K^a_2)\) and \(d - sc(K^b_2)\) we obtain the aggregate social demand \((d - sc)^a + b\) for characteristic 2. In general the unit taxes levied from a and b – \(t^b = sc^a(K^a_2)\) and \(t^a = sc^b(K^a_2)\) – may differ and the Pareto optimum would not be reached with a general tax on the consumption of characteristic 2.

II. D. A Note on the Public Supply in the Lancaster Model

Even if the usual ‘ad hoc’ solution has been avoided by Lancaster, a new possible source of misunderstanding could have been introduced. In fact, there is no reason to prefer the subsidy solution to the public provision, at least in the ambit of the previous analysis, dealing with non excludable goods. In this paragraph we do not aim to solve the general problem of the provision of impure public goods, hence we do not claim to give a final definitive answer, but just to partially vindicate the public provision solution in the Lancaster model.

Naturally things may become quite different (and perhaps much more complicated) in presence of distortionary taxation (introducing the cost of public funds), or explicitly taking into account the different agency problems faced by a private firm and a public firm, or when the exclusion possibility is a feasible option. In this latest case Walsh [1991] probably represents the ultimate reference as he considers a variety of cases and summarises the existing literature, even if it “concentrates entirely on the market provision of (pure) public goods” as West [1991] puts it. But even Walsh [1991] avoids the comparison of private and governmental provision and his prediction that the zero output of public good is not a reasonable long run equilibrium is quite obvious with the Lancaster impure good. Thus we feel in part justified to restrict our analysis to the situation examined by Lancaster, giving a very limited contribution to this controversial area.

We have shown how in general, even in the simple Lancaster model, the private provision to be fully efficient needs to be supplemented by subsidies which are generally of different (per unit) amounts, depending on the household who consumes the impure public good (once we allow for different utility functions and incomes). Now this is in practice a quite complicated business, one may even argue against
the subsidy solution as it can be a vehicle of unjustified discriminations (through the political process), however it is interesting to note that the non-discriminatory public provision solution may be optimal on efficiency grounds, contradicting Lancaster's conclusion.

Lancaster's argument, based on a minimisation of production cost that leaves consumers on the same indifference curves, is shown in Fig. 4. With public provision $X_i$, the impure public good becomes $X_{G}$ a pure public good. Consequently the characteristic transformation curve rotates from $CTC$ (with slope $t = T_2/T_1 - a$) to $CTC_g$ (whose slope is just $T_2/T_1$). To maintain the previous utility levels, for each individ-

![Figure 4](image)

ual, more resources are needed so that the transformation curves should shift upwards as the slope has increased (in absolute value), till the new tangency condition is met.

This happens for $(Z^o, z^o)$ the new optimum and implies a reduction of the public characteristic, due to the substitution effect as the consumers stay on their original indifference curves. Furthermore, it implies a cost increase AF in terms of the private characteristic (or in terms of the private input $x$) and of the original transformation curve with the impure public good. This is also shown in the other graphic on the right where the marginal cost of the same bundle of private and public characteristics ($aX, X$) with the public provision has become $CM_g$ and the equilibrium level of the hypothetical aggregate demand for a bundle having the same characteristics of the impure public good is reduced from $X^*$ to $X^o$.

However, according to the technical specification of the impure public good, it is difficult to understand how it can be supplied by the government as a pure public good, which is the main reason of the peculiar result, as the author explains.\textsuperscript{11} If someone may consume the private characteristic $aX$ of the impure good $X$, one is left with a redistribution problem of the private characteristics of $X_G$. In any case,

\textsuperscript{11} In fact Lancaster argues: "All the private characteristic must be obtained from the private good, partly at the expense of the public good and thus of the public characteristic" (p. 133).
there is no clear reason why the government should behave so irrationally, excluding individual citizens from the consumption of the private characteristic of the impure good; a Pareto worsening policy that implies a malevolent government.

Eq. (2) that defines the private characteristic consumed by each individual \( z_i = x_i + aX_i \) (or the original (8-2)) should then be re-formulated. \( X_G \) may be distributed in equal amount to consumers free of charge by the government. So, under the equal distribution hypothesis, one may write:

\[
(2') \quad z_i = x_i + a(X_i + X_G/n) \quad (i = 1, \ldots, n)
\]

In this case the condition of the marginal rate of substitution MRSC that characterises the first best optimal solution PUB' with government intervention becomes:

\[
(6') \quad T_2/T_1 = a + \sum_1^n \text{MRSC}_i \quad \text{(PUB')}
\]

Comparing it with the optimal solution we find that each individual consumes the same amount of the private and mixed good in both cases. This happens because the two transformation curves CTC and CTC\(_g\) do not differ any longer.\(^{12}\)

\[
(8') \quad \sum_1^n \text{MRSC}_i \text{ (PUB)} = \sum_1^n \text{MRSC}_i \text{ (OPT)}
\]

One possible justification of the previous peculiar result, obtained by Lancaster [1976], lies in the assumption that with public provision consumers should use a different production process. Thus, public provision creates a public intermediate bad which affects the household's feasible production set. In other words, due to its presence the previous process - which transforms a unit of \( X \) into 'a' units of \( z \) and one unit of \( Z \) - is no longer feasible.\(^{13}\) Thus, public inputs and factors may easily occur in this framework and this problem should be properly addressed taking into account analytically De Viti De Marco's emphasis on how the public sector takes part in the exchange process by buying and selling private commodities, transforming these into public goods, which in turn come to influence private processes of production, exchange, and consumption.\(^{14}\)

\(^{12}\) Nothing changes if only government employees have the privilege to consume the private characteristics of the impure good. In fact in his analysis Lancaster just minimises \( T \) subject to \( u_i = u^* \) (for all \( i = 1, \ldots, n \)), given the levels of utilities.

\(^{13}\) For an analysis of this type of bads with linear production systems cfr. Vagliasindi [1994].

\(^{14}\) A number of writers like Boardway [1973] derived efficiency conditions for the supply of public intermediate commodities similar to Samuelson's summation rule. Henderson [1974] claimed that in the
On the other hand, it may be also possible that the public sector production generates jointly a public intermediate factor which affects positively the households’ production functions. In practice, it becomes feasible a more efficient process, which transforms a unit of $X$ into $a'$ (with $a' > a$) units of $z$ and one unit of $Z$. It seems justified to argue that public inputs (jointly generated by the public production sector) are probably positive rather than negative at least in a theoretical framework of interactions between the public and private sectors. But, we leave this general problem to other students, satisfied of having revalued the public provision solution for impure public goods.

III. MERIT WANTS, CHARACTERISTICS, AND THEIR USUAL REPRESENTATION

III. A. Merit Wants and Individual Preferences

It is approximately forty years that the concept of merit wants has been introduced in 1957 by Richard Musgrave, a concept which nicely formalises, in the contest of a fully democratic budgetary process, the inadequacy of public finance’s behavioural preference foundations already felt by many scholars. Without going back to Smith’s concept of impartial observer of scholastic (and perhaps Aristotelian) derivation or to ancient scholars, let us point out how in 1943 Cosciani, a great scholar of public finance, distinguished (as Pareto) wishes and interests of consumers building a simple model with antithetical utility functions of citizens and governors, which (differently from an elitist approach) may depict a real democratic decision process with coercive welfare maximisation.

More recently, Richard and Peggy Musgrave [1985, p. 78], define “merit goods as goods the provision of which society (as distinct from the preference of individual consumer) wishes to encourage or, in the case of demerit goods, to deter” a definition substantially subscribed by Musgrave [1987]. This theory tries to describe some cohercitive phenomena present in the contest of a democratic society.¹⁵ Thus genuine “merit-good situations based on communal interest do exist. As distinct from

¹⁵ In fact “by virtue of sustained association and mutual sympathy, people come to develop common concerns. A group of people, for instance, share a historical experience or cultural tradition with which they identify, thereby establishing a common bond”, cfr. Richard and Peggy Musgrave [1985, p. 77]. On the same matter cfr. also R. Musgrave [1959] and J. Head [1974].
abuse by dictatorial or elitist imposition, they should not be omitted from a normative framework".\textsuperscript{16}

It has been argued that they represent just a terminological trap in the context of welfare economics or that there is no place for them when we assume the sovereignty of consumers. But this is not completely true, as shown by the Musgraves: "The doctrine of communal wants has to be viewed with caution, as it may serve as a convenient vehicle for totalitarian regimes. Yet, common concerns and values do exist in a coherent society and its historical tradition".

With the analytical treatment, proposed by authors like Pazner [1972] and Roskamp [1975], the total amount of merit good \( x \), consumed by the collectivity, is introduced in the social welfare function without referring to the individual utilities.

\[
W = W[u^1(x_1, X); \ldots; u^n(x_n, X); x].
\]

Pazner-Roskamp's treatment may even accommodate public goods as merit goods (as required by Musgrave) because we can also write: \( W = W[u^1(x_1, X); \ldots; u^n(x_n, X); X] \).

However, this approach, not mentioned by the Musgraves, has been criticised as arbitrary. This favoured a difficult line of reasoning that we find in Atkinson [1987]: "We should not lose sight of the fact that Musgrave's conception went beyond externalities or imperfect perceptions: He was explicitly entertaining the possibility that social welfare functions may be non-welfarist, as they have later been called by Sen (for example [1979]). Musgrave has doubts about the notion of consumer sovereignty, however widely interpreted, and recognises that we may wish to take account of considerations broader than individual well-beings in reaching social judgements". This is the "multiple preferences" or "higher values" approach to merit wants as Musgrave himself names it.

We too believe that in a realistic analysis it is necessary to distinguish between wishes and interests of the consumer following Pareto, De Viti, Cosciani, and many other distinguished scholars. This line of thought has led Head [1991] to coherently develop a richer welfare framework in the attempt to fully reconcile merit wants with a broadened concept of consumers' sovereignty (allowing for social interactions). Starting from the Pigouvian trichotomy between "desires", "satisfaction", and "real interest" (or welfare), he added to the hierarchy of individual preferences the highest level of "ethical preferences", which reminds us of Arrowian "values". He then

\textsuperscript{16} Cfr. Richard and Peggy Musgrave [1985, p. 79]. Cfr. also Richard and Peggy Musgrave [1982] at p. 86: "Genuine merit goods situations aimed at interference exist, and they do not fit the traditional framework. In part this reflects elitist domination, but in part also reflects the prevalence of communal interests. The fact that such interests are inconvenient to conventional analysis (what does not fit cannot be!) does not disprove their existence".
explains the divergence between these orderings and explores at length the problems connected with endogenous preferences, showing how the merit wants' dimension may greatly enrich the conventional discussion of public policy based on the "thin theory" of externalities (or, we may say, the broader theory of public characteristics). In Vagliasindi [1990] this very same line of reasoning has been already in part followed through a graphical approach which explores some of the issues tied to the distinction between "desires", "satisfaction", and "real interests".

In this note, coherently with our previous remarks on the public characteristic's analysis, it seems more useful to remain instead in the modern theoretical framework of welfare economics, assuming full rationality, and to distinguish the choices of economic agent at different levels (normal everyday life and at a normative-collective level), aimed to improve their individual and collective welfare.¹⁷

III. B. The Merit Characteristics' Model

In this way we remain near to the original Cosciani model, providing a possible justification of the distinction of the utility functions of citizens and governors, which is quite coherent with real democratic decision processes. In practice, citizens and governors coincide, as assumed by De Viti De Marco in a fully democratic society, so that we limit to the case in which governors assume a "tutorial" or paternalistic role (probably a marginal one in Cosciani mind), going beyond the assumption of class homogeneity with the use of social welfare function.

This choice makes it also possible to use the previous apparatus, in particular the analysis of Auld and Eden [1990], in order to fruitfully re-consider the problem of merit characteristics. Furthermore, the approach we propose is quite general, because it may be also able to deal with the multiple preferences' issue, as well as with the special veil-of-ignorance's setting proposed by Rawls [1971], in a unified framework.

For simplicity's sake let us introduce the problem in the simple Lancaster original setting and consider only two choice situations: (1) the normal everyday life and (2) the normative-collective one. In the first case, in maximising his utility function, the consumer may find it useful and rational (due to strategic or other considerations, perceived as relevant constraints) not to take (fully) into account some consequences of his choice on the value of the utility function. However, when the very same consumer tries to formulate objective evaluations on his and others' welfare and eventual normative judgements at a collective level, he does not feel those constraints.

¹⁷ This idea is also expressed in Margolis [1954]: "Not only are the rules of behaviour of individuals different according to whether they are operating in the everyday individualistic activities or as a conscious part of a social group but also the criteria that are applicable to these two situations are different. To explain the existence of public activities and to evaluate the efficiency of an allocation of the public budget we must refer to the structure of social values."
In analytical terms we may start from the simple utility function like:

\[ v^i = v^i(z, Z; \beta) \]  \hspace{1cm} (14)

where the parameters contained in vector \( \beta \) show the different levels (\( \beta' \) normal everyday life and \( \beta^o \) normative-collective level) at which the agent operates. Referring to these two hypothetical levels we may derive two different utility functions that we may call the everyday-choice utility function:

\[ u^i = u^i(z, Z) = v^i(\beta') \]  \hspace{1cm} (15)

and the optimal-value utility function:

\[ v^i = v^i(z, Z) = v^i(\beta^o). \]  \hspace{1cm} (16)

This second setting may eventually be quite consistent with the framework assumed by Rawls [1971], where the agents do not exactly know their social status when they are deciding the social welfare function to be maximised by the community, or to the Kantian imperative moral choice setting. However, we do not necessarily need to achieve these very high standards in any individual, when he confronts himself with the normative-collective choice situation.

For the sake of the argument let a social welfare function exist and be the usual type in relation to two agents \( a \) and \( b \):

\[ W = W(v^1, ..., v^n). \]  \hspace{1cm} (17)

If we assume that, while the everyday-choice utility function does not depend on the total level of consumption of the characteristic \( z \), the optimal-value utility function does (independently from the identity of single consumers). We can represent it analytically with the following function:

\[ v^i = v^i[u^i(z, Z); z] \]  \hspace{1cm} (16')

where \( z = \sum z_i \). We may thus write the social welfare function as:

\[ W = W[v^1(z, Z; z), ..., v^n(z, Z; z)] = W[u^1(z, Z), ..., u^n(z, Z; z); z]. \]  \hspace{1cm} (18)

This interesting example depicts the case of an aggregate private merit characteristic \( z \), because in the normative-collective situation each agent gives a positive value to the aggregate amount of the private characteristic \( z \), independently from the identity of single consumers. Clearly, the model may deal as well with public merit characteristic
When $v^i = v^i[u^i(z_1, Z); Z]$. In fact, in this case, we would have obtained: $W = W[u^1(x_1, Z); ...; u^n(x_n, Z); Z]$ instead of (18).

With this specification one may take into account many of the arguments contained in Sen [1985]. If, referring to (18), we restrict to the simplest consumption technology, with a single private and public good $x$ and $X$, we have $W = W[u^1(x_1, X), ..., u^n(x_n, X); x]$. This will provide an analytical justification of the usual analytical treatment criticised by NG [1983], because the social welfare function is rigorously derived from the individual optimal-value utility function. Naturally, we do not pretend the Pazner-Roskamp analytical treatment to be the general one but just a particular case of merit goods.\textsuperscript{18} This time the usual subsidy-scheme applies perfectly, thus paradoxically the solution of this case may be even easier at a policy level.

\textbf{FIGURE 5}

Let us illustrate the optimal equilibrium for this case. Each agent values the benefit connected with $x_i$ through his everyday-choice utility function $u^i$ and formulates his private demand $D^i$. Then a merit-valuation of all agents for the meritorious characteristic of his consumption $M(x_i)$ is added, and in equilibrium the subsidy is equal to this valuation. The case represented in Fig. 5 is not the only possible one. However, the graphical solution may be used for illustrating more complex cases, because it allows for different subsidies for each $X_i$.

\textsuperscript{18} The framework proposed is different from the one due to Ng [1983] with the distinction between preferences and welfare on the basis of his social welfare function. His solution is perhaps not very relevant at an operative level. However, we do agree with some of his general statements: "Some economists have difficulty in seeing the above distinction between preferences and welfare saying that whenever an individual prefers x to y, he must be, or at least believes himself to be happier in x than in y. This difficulty completely baffles me. Clearly, a father (or mother) may sacrifice his (her) happiness for the welfare of his (her) children. I cannot see why similar sacrifices cannot be made for a friend or a relative, and further for a countryman, any human being, and finally any sentient creature" (p. 8). For some remarks on the utility interdependence cfr. also Roskamp [1975].
III. C. Toward a Comprehensive Approach to (De)merit Wants

Hence we may claim that the model of Auld and Eden [1990] may be interpreted as a merit wants’ analysis. In fact, it is fully appropriate to represent the situation where $K_k$ represents a pure aggregate demerit characteristic and need not to be modified under any respect. Furthermore, their model seems quite relevant in order to fruitfully re-consider the problem of merit characteristic.

In fact, given the assumption that each household ignores the effects of his choice on the aggregate level ($\Sigma_i K^i_k$) of the $k$-th characteristic in solving his maximisation problem, sticking to Auld and Eden we have implicitly chosen the following everyday-choice utility functions:

\begin{equation}
(15A) \quad u^i = u^i(K^i_1, K^i_2) = v^i(\beta')
\end{equation}

and normative-collective utility functions:

\begin{equation}
(16A) \quad v^i = v^i[u^i(K^i_1, K^i_2); \Sigma_i K^i_k] = v^i(\beta^o).
\end{equation}

Clearly, this new interpretation would not substantially change the original results, and the geometrical representation does not need to be modified.

However, this characteristics’ approach to merit wants is an interesting generalisation with respect to the original treatment of aggregate consumption merit wants due to Pazner and Roskamp. In fact, the merit want does not coincide with any private or public good or even a private or public characteristic, but constitutes a distinct jointly produced characteristic, whose output is generated from the household’s consumption technology.

In this way we may examine a more general model even with respect to the very simple one previously examined. In fact, in general, one can consider $n$ consumers $J$ goods ($X_j$) and $k$ characteristics ($K_l$) of any type, even if one may concentrate on the case of three characteristics ($1$ and $2$) private, $k$ demerit) in order to use their nice geometrical interpretation.

The fact that each household ignores the effects of his choice on the aggregate level of the demerit characteristic ($\Sigma_i K^i_k = \Sigma_i \Sigma_j b_{ij} X^i_j$) is no longer an ad hoc simplification, absurd or unacceptable on methodological grounds, even in the basic two consumers’ model.

Consequently, the equal tax or subsidy on each good represents the optimal policy rule with any number of goods and consumers.

Furthermore the general approach previously proposed is very flexible and may constitute a useful unified framework in which most interpretations of the (de)merit valuations of goods or consumption activity, which involves norms different from restricted consumer sovereignty, may be adequately analysed in the contest of an enlarged social welfare theory.
In fact, our declared starting point was Cosciani's model with antithetical utility functions of citizens and governors, but as previously shown the model can adequately represent what Musgrave [1987] calls the "multiple preferences" approach to merit wants, considering for instance a Kantian imperative moral choice setting or a Rawlsian veil-of-ignorance situation.

We may easily examine also many other different cases. For instance, there is no reason why, instead of representing governors' valuations of utility functions, eq. (16) may not represent community preferences accepted by the consumers, even if different from their personal preferences (15), because they reflect common concerns and values, deeply rooted in the historical tradition of a given society. But this is just what Musgrave calls the "community preferences" approach to merit wants.

Finally, referring back to the initial very simple basic model, let us marginally modify it in order to represent a case of "paternalism in distribution" approach; to examine a limiting situation let us turn to 'primary distribution' a situation that related to the philosophical concept of 'primary goods' where a minimum provision of these goods must be provided, instead of a minimum income.

To get this result one may postulate that each optimal-value utility increases only when all \( z_i \) become greater than a certain amount, and that this level is established by whom consumes the least. Thus, in this limiting situation, variables \( z_i \) influence the optimal-value utility function only through \( \text{min}_i(z_i) \); that is:

\[
(16') \quad v^i = v^i[z_i, Z; \text{min}_i(z_i)].
\]

We may thus write the social welfare function as:

\[
(18'') \quad W = W\{u^1(z_1, Z; \text{min}_i(z_i)), ..., u^n(z_n, Z; \text{min}_i(z_i))\} = W[u^1(z_1, Z), ..., u^n(z_n, Z; \text{min}_i(z_i)].
\]

Thus, assuming that \( m \) has the lowest level of \( z \), and remains so in the optimal situation, \( z_m \) is the merit characteristic and has a role similar to a public good.\(^{19}\) In this simple way equity considerations appear and become dominant in this model of merit wants.

The different situations may be easily represented in the case of two consumers \((i = a, b)\), considering the \((z_b, z_a)\) space. With eq. (18) each individual is not interested in the way in which the aggregate level \( \Sigma z_i \) is distributed among different individuals so that (ceteris paribus) social welfare functions may look as a straight line. In the case of eq. (18'') instead individuals care about an egalitarian distribution of the private characteristic \( z \) among the members of the community, and so the curves are L-shaped. In the bi-dimensional case assuming a community of two individuals \( a \) and

\(^{19}\) This may help to understand some remarks present in Tiebout and Houston [1962] about merit goods.
b one may draw not only other intermediate curves \( i \) between \( u \) and \( e \) but may allow even for ultra-egalitarian views \( ue \).

In general each individual consumption \( z_i \) may enter the optimal-value utility function like in the following equations, with may represent the most general formulation.

\[
(16^*)
\]

\[
v^i = v^i(z_1, \ldots, z_n, Z).
\]

However, even in the most simple case \((16')\) the previous analysis does not solve completely the real problems of collective choice and of optimal provision of merit wants. Nevertheless in this way we have shown how it is not fully correct to start from the everyday-choice utility function (derived from consumer explicit choices or revealed preferences) in order to discuss welfare and public good problems in reality and we have also re-used a traditional model on a more realistic base of collective choice.

III. D. Further Reflections on the Implementation Issue for Merit Characteristics

We have seen how, once we allow for different utility functions and incomes, apart from particular simple models (the cases of an aggregate private merit characteristic or of a public merit characteristic), to reach optimality (even only on efficiency grounds) we need a quite complicate tax-subsidy scheme, with different rates for each household, which can lead to unjustified discriminations (through the political process), as in the Lancaster impure public good. This seems the main reason that has led West [1991] to embrace, once again, the thesis of an empty box, attempting to stop further developments of Head's analysis of merit wants. The main argument may be phrased as follows. Why should we bother with merit wants, if the "residual" governmental activities (that cannot be explained by public characteristics) are perfectly explained in terms of "newer economics" of bureaucracy and rent-seeking?
In substance West believes that the “benign rationalisation” of these residual activities in terms of merit wants introduces only new possible sources of inefficiencies (like with public health and education). In what follows, we will just try to provide a first line of defence of the theory of merit characteristics, on this more practical but nevertheless relevant ground, leaving any definitive answer to the general implementation issue to other contributions.

First of all, it should be recognised that even the absence of any real merit wants policy does not demonstrate that it is not needed, as the existence of a dictatorial regime is not a compelling argument against the need for liberty. Clearly, this is an important point for the ones interested in a purely normative analysis. For instance, Samuelson’s classical contribution “A pure theory of public expenditure” went beyond this point only to say that the optimal solution could not be reached by a private “decentralised pricing system” or a political process (apart from special situations as the Kantian imperative moral choice setting and we may add the Rawlsian framework).

No surprise that the arguments and evidences that West (and other subscribers of the government failure) may produce are difficult (even if not always impossible) to falsify. However, we do not need to enter the stage of empirical evidence, on this very controversial issue, as at the present we are just interested in the relevance of Head’s (and ours) analysis of merit wants and not on the practical implementation issue in itself.

In reality, the government failure is not limited to merit wants but it involves any governmental activity; no particular reason has been given to suppose that it should affect more deeply the implementation of merit wants policy (rather than social ones). Only in pure theory we may disregard the effects of the all pervasive presence in the public sector of distortionary taxes and agency problems.

Why then should not all governmental activities (not only merit wants policy) be prevented? Simply because, as any trained economist would say, we may find an optimal trade-off considering explicitly inefficiencies as an additional cost. We may also try to control directly the emergence of rent-seeking expenditures with more careful public estimates of who gets the benefits (by profession and income category) of public expenditure programs.

Hence, West is quite right in warning us against the danger of using the pure theory to “rationalize every aspect of government intervention” (a danger of which Head seems quite conscious) but from this it does not follow that the merit want is an empty concept otherwise one would simply disregard all the pure theory (and why not all geometry as triangles do not exist in real world). Of course we should carefully keep watching over the institutional designs.

Remaining in a fully democratic context, let us introduce part of a more common interpretation of Cosciani’s model, contemplating a more coercive context, where governors impose their own valuation, differently from the previous optimal-value
social welfare function. This may be represented in analytical terms introducing a
new decision setting associated with the particular level $\beta^*$ (of eq. (14) different from
$\beta^t$ and $\beta^0$) at which the governors operate. In practice, for the very same consumer
(when he operates as the governor) it is useful and rational (due for instance to specific
institutional designs) to consider some further practical consequences connected with
his particular position and the normal behaviour of his fellows, as they are not living
in a Kantian or Rawlsian choice setting.

Let us make just an example to partially clarify matters. Considering again the
Auld-Eden model, starting from equation:

$$(16A^*) \quad v^i = v[u^i(K^i_1, K^i_2); \zeta \Sigma_i K^i_k] = v^i(\beta^*)$$

with $0 < \zeta < 1$ and $v$ an average function, the welfare maximisation would probably
lead to a reduced total level of the merit characteristic, with respect to the optimal one
obtained from (16A). However, the new equilibrium may actually represent something
near to the second best level, once we take into account also the burden of taxation
and the additional costs and distortions due to the presence of rent-seeking activities.

We will not go further on these lines of reasoning but these types of arguments are
quite obvious (if one refers to Cosciani and the Italian tradition) and fully economically
justified in the context of the recent institutional incentive design theory, as developed
for instance in Stiglitz [1989], or in Laffont and Tirole [1990]. As we have seen these
arguments may also have major consequence on the second best provision of public
and merit characteristics.

IV. CONCLUDING REMARKS

In this work we have tried to re-consider the concept of public and merit goods by
means of the characteristics’ analysis approach. Given the fact that in this field it is not
possible to do full justice to all existing contributions, it has been found more useful to
concentrate on simple models, like the one proposed by Lancaster in 1976 and by Auld
and Eden in 1990. A first result consists in a correct formulation of impure public
good pseudo-demands and in a closer comparison between two possible solutions:
subsidy-scheme and public provision. Furthermore, this analysis allows also to deal
with public intermediate commodities at the individual consumer’s level (following
the hints of De Viti De Marco, Pigou) and to re-consider the problem of merit wants,
justifying and extending the analytical treatment of merit goods followed by Pazner
[1972] and Roskamp [1975].

In this paper we have illustrated some theoretical schemes, that may be found useful
in order to put the problem of public and merit wants in a proper perspective and
to provide correct policy solutions. Even if the hypotheses are quite restrictive this
approach, which explicitly deals with different processes, may be closer to reality and
more useful than the conventional one. We are aware of the fact that improvements and further researches are required but we hope that this perspective will help to throw some new lights on the subject so that students interested in the field will find some useful hints and starting points for future developments.

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